INCIPIENT FLUIDIZATION VELOCITIES OF POLYDISPERSE MATERIALS

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Equations are presented for calculation of velocities at incipient fluidization (minimum-fluidization velocity) of polydisperse (multi-size particle) materials. They are simultaneously transformed into relations between various arguments which can be primarily given as the system characteristics. The basic information on the particle bed is the sieve analysis. In solving the problem, it was determined that until now the most exact relations for velocity at incipient fluidization of single-size particle (monodisperse) beds start to be affected by an error at values of $Ar > 2 \cdot 10^7$, which error quickly increases with the value of Ar. A suitable equation has been calculated from the experimental data. From the relation between Re_p and Ar for monodisperse beds of spherical particles follows that at $Ar > 2 \cdot 10^7$ neither the expansion equations nor relations for the friction factor of monodisperse uniformly fluidized beds can be correct.

We have shown in our previous works[1-4] that for calculation of the minimum fluidization velocity of polydisperse materials two types of beds must be distinguished: the beds without a segregation region[1,2] and beds with a segregation region[3,4]. This paper as an evaluation of experimental data, offers a widely valid calculation relations for beds with the segregation region and new information on friction factor for flow in a uniformly fluidized bed. If for the velocity at incipient fluidization of single-size particles holds the relation

$$Re_p = f(Ar)\,, \tag{1}$$

then for polydisperse beds with the segregation region holds the relation[3]

$$Re_{pCZ} = f[(1 - y)\,Ar_C]\,. \tag{2}$$

Index C denotes that for the characteristic length in the Reynolds and Archimedes numbers should be substituted the so-called representative diameter of the fraction of coarse particles defined as: mixture of particles of various sizes in a bed is considered to be a binary mixture of coarse and fine particle fractions. The fraction of coarse particles includes all particles with the size in range

$$0{\cdot}5d_{\max} \leqq d_i \leqq d_{\max}\,. \tag{3}$$

The remaining particles form the fraction of fine particles. Both fractions are considered to be independent units ($\sum x_i = 1$ holds for each unit separately) and the representative diameter d_C, resp. d_F is then

$$d_C = 1/\sum(\bar{x}_{iC}/d_{iC})\,, \tag{4a}$$

resp.

$$d_F = 1/\sum(\bar{x}_{iF}/d_{iF})\,. \tag{4b}$$

The quantity y was defined[3] as the ratio $y = \Delta p^*/\Delta p_t$, where Δp^* is the surplus drop of static pressure in the so-called last fixed bed of coarse particles at incipient fluidization region caused by pulsation of particles fluidizing above this bed and Δp_t corresponds to the effective weight of the last fixed bed per unit of cross-sectional area. The value y can be under special condition measured or determined indirectly – by calculation from equations of type (2). Reliable direct measurements can be made only with binary mixtures. Thus we have chosen the indirect method. Additional experimental data given in Tables I and II were added to those listed earlier[3]. In selected cases, the value y was also determined by direct measurement and the result was in a very good agreement with the value calculated by the indirect method.

TABLE I

Properties of Narrow Fractions of Spherical Particles Used for Measurement of Velocities at Incipient Fluidization of Binary Mixtures

Data are related to those of paper[3].

Material	d_i, cm	Density ϱ_s, kg/m^3	Material	d_i, cm	Density ϱ_s, kg/m^3
B_{24}	0·0105	2 723·2	B_{29}	0·1452	2 849·1
B_{25}	0·0176	2 941·9	B_{30}	0·2306	2 512·2
B_{26}	0·0343	2 677·7	B_{31}	0·5088	2 504·4
B_{27}	0·0430	2 983·0	B_{32}	0·8065	2 451·9
B_{28}	0·0740	2 945·1			

Selection of Relations for Calculation of Quantity y from Experimental Velocities at Incipient Fluidization

Sufficiently exact relations of the type (1), resp. (2) and experimental data of velocity at incipient fluidization are required for indirect determination of quantity y. Since the function in relation (2) has the same characteristics as in relation (1), the reliability of calculated values y will be ensured when a sufficiently accurate equation is chosen for calculation of velocity at incipient fluidization of monodisperse beds. While in the years 1948 to 1959 twenty such equations were found in literature[5], Romankov and Raškovskaja[6] present for years 1948 to 1962 already 37 equations, the book by Aerov–Todes[7] gives a table with 76 equations. As can be seen from the work by Wen and Yu[8], neither this number is complete.

For calculation of velocities at incipient fluidization of monodisperse beds we considered as suitable the relations of type (1) as follows

$$\mathrm{Re_p} = 0{\cdot}00138\ \mathrm{Ar}/(\mathrm{Ar} + 19)^{0{\cdot}11}\,, \tag{5}$$

TABLE II

Velocities at Incipient Fluidization of Binary Mixture Experiments
Data are related to those of paper[3].

No	Material	$\bar{x}_F$	M g	i °C	P Torr	$\mathrm{Ar_C}$	z	$\mathrm{Re_{p\,exp}}$
107	$B_{26,24}$	0·5	400	18·5	750	3 872·9	7·134	1·15
108	$B_{27,25}$	0·5	500	18·5	750	8 495·5	4·436	2·32
109	$B_{28,25}$	0·5	500	18	758	43 978·5	10·975	8·70
110	$B_{28,26}$	0·5	500	18	758	43 978·5	3·931	11·45
111	$B_{29,26}$	0·5	400	17	749	322 080	8·539	43·08
112	$B_{29,27}$	0·5	400	17	749	322 080	5·317	43·38
113	$B_{29,28}$	0·5	400	17	749	322 080	2·172	49·19
114	$B_{30,25}$	0·5	400	17	749	1 105 000	41·589	117·10
115	$B_{30,26}$	0·5	400	18·5	749	1 105 000	11·512	118·46
116	$B_{30,28}$	0·5	400	18·5	749	1 105 000	2·931	111·85
117	$B_{31,27}$	0·5	400	20	750	11 821 000	13·636	491·93
118	$B_{31,28}$	0·5	400	20	750	11 821 000	5·570	485·54
119	$B_{31,27}$	0·5	500	18·7	749	11 833 000	13·633	481·47
120	$B_{31,28}$	0·5	500	18·7	749	11 833 000	5·569	464·63
121	$B_{31,29}$	0·5	500	18·7	749	11 833 000	2·542	521·80
122	$B_{32,28}$	0·5	400	20·5	750	45 746 000	7·990	1 067·30
123	$B_{32,29}$	0·5	400	20·5	750	45 746 000	3·638	1 325·59

for

$$\mathrm{Ar} \leqq 1{\cdot}06 \,.\, 10^5 \,, \tag{6a}$$

resp.

$$\mathrm{Re_p} \leqq 41{\cdot}0 \tag{6b}$$

and for the case of inversed inequalities

$$\mathrm{Re_p} = 0{\cdot}03865 \; \mathrm{Ar}^{0\cdot602} \,. \tag{7}$$

Validity of Eq. (7) was verified up to values of $\mathrm{Ar} = 2{\cdot}13 \,.\, 10^8$ resp. $\mathrm{Re_p} = 3{\cdot}99 \,.\, 10^3$. Eq. (5) and (7) were calculated under the assumption that for spherical particles it holds

$$\varepsilon_p = 0{\cdot}420 \,. \tag{8}$$

Condition (6a) resp. (6b) was obtained by solution of Eq. (5) and (7) for an unknown $\mathrm{Re_p}$ and Ar.

Eq. (5) may be transformed into the relation

$$\mathrm{Re_p} = (0{\cdot}000611 \; \mathrm{Ar})^n \,, \tag{9}$$

wherefrom equality of right hand terms of Eq. (5) and (9) follows

$$n = \log\left[0{\cdot}00138 \; \mathrm{Ar}/(\mathrm{Ar} + 19)^{0\cdot11}\right]/\log\left(0{\cdot}000611 \; \mathrm{Ar}\right) . \tag{10}$$

The constant 0·000611 was chosen so that at greater Ar numbers in agreement with the experiment holds

$$n \approx 0{\cdot}89 \,. \tag{10a}$$

A limit $n = 0{\cdot}8913$ of the right hand side of relation (10) exists simultaneously if $\mathrm{Ar} \to 1/0{\cdot}000611 = 1637$. Value n as a function of Ar can be read off from Fig. 1 which represents Eq. (10). From this graph can be seen that for all values of $\mathrm{Ar} > 200$ can be used relation (10a). At $\mathrm{Ar} \leqq 7{\cdot}2$ it is more correct to use the relation[9,10]

$$\mathrm{Re_p} = 0{\cdot}0009836 \; \mathrm{Ar} \,. \tag{10b}$$

For polydisperse beds with the segregation region are from equations (5), (7) and (9) obtained[3] relations of the type (2):

$$\mathrm{Re_{pCZ}} = 0{\cdot}00138(1 - y)\,\mathrm{Ar_C}/[(1 - y)\,\mathrm{Ar_C} + 19]^{0\cdot11} \,, \tag{11}$$

resp.

$$\mathrm{Re_{pCZ}} = [0{\cdot}000611(1 - y)\,\mathrm{Ar_C}]^n \,, \tag{12}$$

for

$$(1 - y)\,\mathrm{Ar} \leqq 1{\cdot}06\,.\,10^5\,, \tag{13a}$$

resp.

$$\mathrm{Re}_{\mathrm{pCZ}} \leqq 41{\cdot}0 \tag{13b}$$

and for the case of inversed inequalities (*13a*) resp. (*13b*) the relation

$$\mathrm{Re}_{\mathrm{pCZ}} = 0{\cdot}03865[(1 - y)\,\mathrm{Ar}]^{0{\cdot}602}\,. \tag{14}$$

With respect to verification of Eq. (7), the validity of Eq. (*14*) can be considered verified up to values $(1 - y)$ Ar $= 2{\cdot}13\,.\,10^8$, resp. $\mathrm{Re}_{\mathrm{pCZ}} = 3990$.

Eq. (*5*) is calculated from our numerous measurements of expansion of uniformly fluidized beds in a wide range of values of Ar number with sufficient accuracy. It agrees with the characteristic course of expansion curves in the regions which we denoted[9,10] as laminar and pseudolaminar. When we choose as a characteristic length of non-spherical particles the diameter of a sphere of equal volume

$$d = (6V/\pi)^{1/3}\,, \tag{15}$$

then Eq. (*5*) and limits of its validity do not depend on the particle shape. A more exact limit of its validity, which at the same time determines boundary of the pseudolaminar region, is Ar $= 7{\cdot}2\,.\,10^4$ resp. $\mathrm{Re}_\mathrm{p} = 29$. Its transfer to the level of conditions (*13a*) resp. (*13b*) is the cause of increase in values of Re_p within the range of up to 7%.

Few data are available on expansion of monodisperse beds of spherical particles and on velocities at incipient fluidization for Ar $> 10^5$. Eq. (*7*) was, therefore, calculated from our experimental data on velocities at incipient fluidization of spherical particles given in Table III. Design parameters of the column with the used measuring method are given in paper[3]. The height of fixed bed was verified and its porosity was $\varepsilon_\mathrm{p} = 0{\cdot}420 \pm 0{\cdot}004$. The range of particles sizes in the bed was less than that limit-

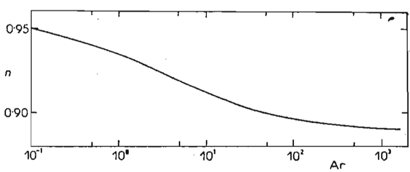

FIG. 1

Dependence of Exponent *n* in Eq. (*9*) on Archimedes Number ed by dimensions of holes of consecutive standard sieves DIN and is typically represented by the histogram for ceramic particles in Fig. 2. The measured data fit very well the straight line of log Re_p on log Ar. Furthermore, they completely correspond with data by Wen and Yu[8].

Comparison of equations (5) *and* (7). Full line in Fig. 3 represents the dependence of Eqs (5) and (7) plotted through data of 17 authors[11–27]. At high values of Ar our measured Re_p are in a very good agreement with data of Fetterman[25] and at Ar = = $4{\cdot}67 \cdot 10^8$ the resulting values of Re_p from Eq. (7) is 1·67 times greater than the value obtained by measurements of Johanson[23] and Kelly[24]. Both compared groups of data were measured with water and with spherical particles in columns of diameter 10·16 cm, resp. 15·24 and 22·86 cm. An explanation will be given farther in connection with the use of equation derived from the Ergun equation for pressure drop in fixed bed.

TABLE III

Experimental Velocities at Incipient Fluidization of Narrow Fraction of Spherical Particles

No	Material	d_i, mm	ϱ_s, kg/m^3	Ar	$Re_{p\,exp}$	$Re_{p\,calc}/Re_{p\,exp}$
1[a]	glass	2·032	2 450	$1{\cdot}19 \cdot 10^5$	43·05	1·020
2	glass	1·108	2 944	$1{\cdot}44 \cdot 10^5$	44·3	1·112
3	glass	1·209	2 749	$1{\cdot}68 \cdot 10^5$	49·3	1·097
4	glass	1·245	2 973	$2{\cdot}07 \cdot 10^5$	59·55	1·029
5	glass	1·326	2 737	$2{\cdot}23 \cdot 10^5$	59·8	1·072
6	glass	1·351	2 972	$2{\cdot}52 \cdot 10^5$	68·4	1·009
7	glass	1·452	2 849	$3{\cdot}22 \cdot 10^5$	75·3	1·062
8	glass	1·645	2 901	$4{\cdot}51 \cdot 10^5$	110·5	0·886
9	glass	1·732	2 862	$5{\cdot}25 \cdot 10^5$	108	0·994
10	glass	2·071	2 572	$7{\cdot}98 \cdot 10^5$	145	0·952
11[a]	steel	2·380	7 840	$8{\cdot}95 \cdot 10^5$	148·5	0·996
12	glass	2·306	2 512	$1{\cdot}105 \cdot 10^6$	183	0·918
13	glass	2·460	2 522	$1{\cdot}29 \cdot 10^6$	198	0·931
14[a]	glass	5·004	2 460	$1{\cdot}79 \cdot 10^6$	219	1·026
15	glass	2·739	2 527	$1{\cdot}83 \cdot 10^6$	240·5	0·946
16	glass	3·087	2 567	$2{\cdot}67 \cdot 10^6$	292	0·978
17[a]	glass	6·350	2 360	$3{\cdot}44 \cdot 10^6$	331	1·005
18	glass	5·088	2 504	$1{\cdot}18 \cdot 10^7$	687	1·017
19[b]	glass	8·065	2 452	$4{\cdot}57 \cdot 10^7$	1 600	0·987
20[b]	ceramics	13·947	2 225	$2{\cdot}13 \cdot 10^8$	3 790	1·053
21[b]	ceramics	13·947	2 225	$1{\cdot}83 \cdot 10^8$	3 360	1·083

[a] Data according to Wen and Yu[8]. [b] Data measured in column of diameter 110·1 mm, other data in column of diameter 60·0 mm.

The suitability of equations can be according to Fig. 3 compared only qualitatively as the used data include particles of various shapes, their characteristic length is defined differently and the fraction width (monodispersity) is not limited in the same way.

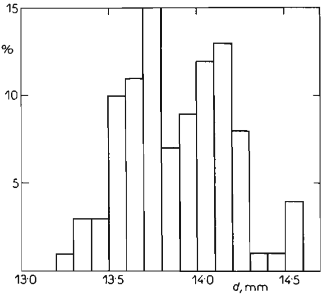

FIG 2

Histogram of Particle Distribution (Ceramic Particles, Characteristic Diameter d 13·947 mm; $\varrho_s = 2265{\cdot}0\ \mathrm{kg/m^3}$) According to Diameters d_i

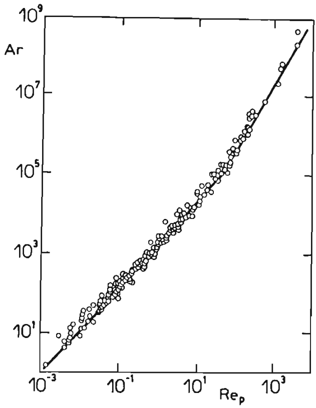

FIG. 3

Comparison of Eqs (5) and (7) with Re_p Values of Different Authors
—— Eq. (5), resp. (7); ○ data of authors[11–27].

Eq. (7) is contradictory to the present state of knowledge on expansion of uniformly fluidized beds as it is obvious from the following consideration: if Eq. (7) is correct then the various relations (incl. ours) are not generally valid for expansion of uniformly fluidized beds at conditions which we denoted[10] as transitional and turbulent regions and which can be arranged into equivalent relations

$$w/u_t = F_1(\varepsilon)\,, \tag{16}$$

$$\mathrm{Re}/\mathrm{Re}_t = F_2(\varepsilon)\,, \tag{17}$$

$$\xi/\xi_t = F_3(\varepsilon)\,. \tag{18}$$

Eqs (*16*)–(*18*) are equivalent to the hypothesis that at ε = const. is in transitional and turbulent regions the dependence $\xi = \mathrm{f}_1(\mathrm{Ar})$ in the plot parallel to the dependence $\xi_t = \mathrm{f}_2(\mathrm{Ar})$.

If Eqs (*16*)–(*18*) are valid, then by relating the incipient fluidization region to $\varepsilon_p = 0{\cdot}420$ or to another value independent of the particle size (or of Ar number, *etc.*), for the velocity at incipient fluidization of monodisperse beds without channelling should generally hold

$$w_p/u_t = \text{const.}, \tag{19}$$

$$\mathrm{Re}_p/\mathrm{Re}_t = \text{const.}, \tag{20}$$

resp.

$$\xi_p/\xi_t = \text{const.}, \tag{21}$$

as is assumed by the majority of authors.

These relations are contradictory to Eq. (7) as can be seen from Table IV. In this Table, the given ratios $\mathrm{Re}_p/\mathrm{Re}_t$ were obtained so that for Ar number was taken the

TABLE IV

Dependence of Ratio $\mathrm{Re}_p/\mathrm{Re}_t$ on Ar Number for Spherical Particles

Ar	$\mathrm{Re}_p/\mathrm{Re}_t$	Re_t	Ar	$\mathrm{Re}_p/\mathrm{Re}_t$	Re_t
103 125	0·0806	500	14 332 500	0·1122	7 000
183 750	0·0815	700	30 000 000	0·1226	10 000
345 000	0·08335	1 000	135 000 000	0·1516	20 000
1 260 000	0·0909	2 000	317 250 000	0·1690	30 000
2 700 000	0·0959	3 000	918 750 000	0·1923	50 000
7 125 000	0·1032	5 000	1 837 500 000	0·2085	70 000

corresponding Re_t from Perry[28] and Re_p was calculated by Eq. (7). Ratio Re_p/Re_t is a function of Ar and only at Ar numbers of the orders 10^5 and 10^6 it attains the value ascribed to it as to a universal constant (the reason probably is that for greater Ar numbers few experimental data are available). The dependence of Re_p/Re_t on Ar is qualitatively in agreement with data by Denton[31] who stated experimentally that the friction factor decreased monotonically in dependence on Reynolds number in the fixed bed of spherical particles in the whole range of subcritic flow around the spherical particle, *i.e.* up to the value of $Re = 10^5$. Accordingly, the correct extrapolation of Eq. (7) up to $Ar = 10^{12}$ can be expected.

From dependence of ratios Re_p/Re_t on Ar number follows that for expansion of uniformly fluidized beds of monodisperse particles in the transitional and turbulent regions instead of Eq. (16)–(18) the relations are valid

$$w/u_t = F_4(\varepsilon, \mathrm{Ar})\,, \tag{22}$$

$$\mathrm{Re}/\mathrm{Re}_t = F_5(\varepsilon, \mathrm{Ar})\,, \tag{23}$$

$$\xi/\xi_t = F_6(\varepsilon, \mathrm{Ar})\,, \tag{24}$$

where the Ar number can be replaced by Re_t at the changed characteristics of functions F_4 to F_6.

The functions F_4, F_5, F_6 are such that

$$F_i(\varepsilon, \mathrm{Ar})_{\varepsilon=1} = 1\,. \tag{25}$$

We could accept neither of the two equations by Wen and Yu[8] which they considered to be of general validity. One of their equations was derived on the basis of hypothesis which can be formulated as follows:

The ratio of hydrodynamic force (drag force) F_0 by which the liquid acts on the particle in the bed and of force F_s by which the liquid would act in the unlimited space on the particle moving in it with velocity v, equal to superficial liquid velocity w in the bed at otherwise same conditions, is a universal function of porosity

$$F_0/F_s = \mathrm{f}(\varepsilon)\,. \tag{26}$$

We will show that this hypothesis is not equivalent to that of ours on parallelity of curves $\xi = \mathrm{f}_1(\mathrm{Ar})_{\varepsilon=\mathrm{const.}}$ and $\xi_t = \mathrm{f}_2(\mathrm{Ar})$ but the generalized conclusions are not in a agreement with reality. Since in a uniformly fluidized bed of monodisperse spherical particles holds

$$F_0 = (\pi d^3/6)\, g(\varrho_s - \varrho_f)\,, \tag{27}$$

we can for equality $w = v$ with regard to relation (26) and to Newton equation for the drag force

$$F_s = (\pi d^2/4)\, \xi_s(w^2 \varrho_f/2)$$

write

$$\xi_s = \frac{4gd(\varrho_s - \varrho_f)}{3\varrho_f w^2} \frac{1}{f(\varepsilon)}, \tag{28}$$

where ξ_s is a known function $f_s(\mathrm{Re})$, where $\mathrm{Re} = wd\,\varrho_f/\mu$, *i.e.*

$$\frac{4gd(\varrho_s - \varrho_f)}{3\varrho_f w^2} \frac{1}{f(\varepsilon)} = f_s\,(\mathrm{Re})\,. \tag{29}$$

From the definitions of ξ, ξ_t and ξ_s relations follow

$$\xi_t = (\xi_s)_{\varepsilon=1}, \text{ when } f(\varepsilon) = 1 \text{ and simultaneously } w = u_t \tag{30}$$

$$\xi = \xi_s \,.\, f(\varepsilon)\,, \tag{31}$$

and from these we obtain

$$\frac{\xi}{\xi_t} = \frac{u_t^2}{w^2} f(\varepsilon)\,. \tag{32}$$

The right-hand side of Eq. (*32*) is not equivalent to that of Eq. (*18*), therefore neither the corresponding hypotheses are equivalent.

Let us check the correctness of Eq. (*29*) at the assumption that relation (7) is valid. On multiplying by Re^2 and after arrangement we obtain

$$f(\varepsilon) = (4/3)\,\mathrm{Ar}/\mathrm{Re}^2\, f_s(\mathrm{Re})\,. \tag{33}$$

Further with regard to the nature of $f_s(\mathrm{Re})$ it is possible to write

$$\mathrm{Re}^2\, f_s(\mathrm{Re}) = (4/3)\,\mathrm{Ar}^*\,, \tag{34}$$

where Ar^* is read off for value of $\mathrm{Re}_t = \mathrm{Re}$ from the dependence $\mathrm{Re}_t = f_3(\mathrm{Ar})$ for free fall of the particle in an unlimited viscous liquid. From this it holds

$$f(\varepsilon) = \mathrm{Ar}/\mathrm{Ar}^*\,. \tag{35}$$

According to Wen and Yu is $f(\varepsilon) = \varepsilon^{-4\cdot7}$ resp. for $\varepsilon_p = 0{\cdot}420$ should $f(\varepsilon_p) = 59{\cdot}00$. When we read off Ar^* for $\mathrm{Re}_t = \mathrm{Re}_p$ then should be $\mathrm{Ar}/\mathrm{Ar}^* = 59{\cdot}00$. In Table V are given data of $\mathrm{Ar}/\mathrm{Ar}^*$ as a function of Ar. For chosen values of Ar numbers the values Re_p were calculated by Eq. (7) and Ar^* numbers corresponding to them were found by interpolation equations which we have elaborated[29] for accurate interpolation of data by Perry[28].

As can be seen, our measurements do not confirm correctness of the hypothesis (*26*) which should be modified in accordance with the relation

$$F_0/F_s = f_4(\varepsilon, \mathrm{Ar})\,, \tag{36}$$

where

$$f_4(\varepsilon, \mathrm{Ar})_{\varepsilon=1} = 1\,. \tag{37}$$

The Ar number can be at a change of characteristics of f_4 substituted by Re_t or ξ_t.

We could not, therefore, accept the equation

$$\mathrm{Ar} = 1060\ \mathrm{Re_p} + 159\ \mathrm{Re_p^{1\cdot687}} \tag{38}$$

which is correct if hypothesis (*26*) is simultaneously valid with the equation for friction factor ξ_t

$$\xi_t = 24/\mathrm{Re}_t + 3\cdot60/\mathrm{Re}_t^{0\cdot313}\,. \tag{39}$$

Wen and Yu have also modified equation of Ergun and Orning[12,30] for pressure drop in the fixed bed which after substitution of $\Delta p/l = g(\varrho_s - \varrho_f)(1 - \varepsilon)$ and arrangement has the form

$$\frac{1\cdot75}{\varphi\varepsilon^3}\,\mathrm{Re}^2 + \frac{150(1-\varepsilon)\,\mathrm{Re}}{\varphi^2\varepsilon^3} - \mathrm{Ar} = 0\,, \tag{40}$$

into which they substituted for the incipient fluidization region

$$1/\varphi\varepsilon_p^3 = 14\,, \tag{41}$$

$$(1 - \varepsilon_p)/\varphi^2\varepsilon_p^3 = 11\,. \tag{42}$$

By its solution for $\mathrm{Re_p}$ they obtained the relation

$$\mathrm{Re_p} = (33\cdot7^2 + 0\cdot0408\ \mathrm{Ar})^{1/2} - 33\cdot7\,, \tag{43}$$

which should be valid within the whole considered range

$$\mathrm{Re_p} \in \langle 0\cdot001\,;\ 4000\rangle\,,$$

despite of their claims that Eq. (*40*) is valid up to $\mathrm{Re} = 3000$. Eq. (*43*) cannot be accepted for spherical particles because $\varphi = 1$ and if we take $1/\varphi\varepsilon_p^3 = 14$, then $\varepsilon_p = 0\cdot415$ and from that

TABLE V

Dependence of Ratio Ar/Ar* on Ar Number

Ar	$\mathrm{Re_p = Re_t}$	Ar/Ar*	Ar	$\mathrm{Re_p = Re_t}$	Ar/Ar*
$1\cdot06 \cdot 10^5$	40·96	49·69	$1 \cdot 10^7$	632·63	64·13
$3 \cdot 10^5$	76·63	56·20	$3 \cdot 10^7$	1 225·7	59·61
$5 \cdot 10^5$	104·21	58·38	$5 \cdot 10^7$	1 664·2	55·72
$7 \cdot 10^5$	127·61	59·86	$7 \cdot 10^7$	2 041·3	53·30
$1 \cdot 10^6$	158·18	61·47	$1 \cdot 10^8$	2 530·2	50·85
$3 \cdot 10^6$	306·46	66·52	$3 \cdot 10^8$	4 902·0	43·98
$5 \cdot 10^6$	416·80	65·50	$5 \cdot 10^8$	6 667·0	38·69
$7 \cdot 10^6$	510·39	64·83	$7 \cdot 10^8$	8 163·9	35·51

$(1 - \varepsilon_p)/\varphi^2\varepsilon_p^3 = 8.19$ instead of 11, see Eq. (*42*). But if we choose $\varepsilon_p = 0.420$, then

$$1/\varphi\varepsilon_p^3 = 13.5\,, \tag{44}$$

resp.

$$(1 - \varepsilon_p)/\varphi^2\varepsilon_p^3 = 7.83\,, \tag{45}$$

and for the velocity at incipient fluidization we obtain

$$\mathrm{Re}_p = (618.12 + 0.04234\ \mathrm{Ar})^{1/2} - 24.86\,. \tag{46}$$

By use of the Ergun equation *i.e.* as well Eqs (*43*) and (*46*) the following difficulties arise at least:

For a fixed bed of smooth spherical particles ($\varepsilon = 0.42$) is this equation sufficiently accurate only if[7,31] $\mathrm{Re} \leq 1\,740$ and if instead of the constant 1·75 in the first term is used 1·35 and in the second 163·8 instead of 150. With slightly rough spheres, however, the Ergun equation is valid up to $\mathrm{Re} \approx 8.7 \cdot 10^3$. After application of the respective data to fluidization, it follows that at $\mathrm{Re}_p > 1\,740$ the bed of smoothly polished spheres should have a greater value of Re_p than that of slightly rough spheres and these again should have a greater value of Re_p than the bed of very rough spheres. Polishing of the surface of slightly rough spheres should change for example Re from 3000 to 3450. If this is correct, then the differences in values measured by Johanson and Kelly (on one hand) and ours and Fetterman's (on the other) could be caused by different surface roughness. (Our ceramic particles were varnished but not polished; Johanson and Kelly used steel spheres or lead shots). In such case at $\mathrm{Re}_p > 1\,740$ it is necessary to introduce into the calculation equations for velocities at incipient fluidization and for expansion of fluidized beds the roughness of the particle surface. However, this statement should be taken very cautiously and must be further verified since: research of the flow in fixed beds does not take into consideration that the changed character of the flow cannot be expressed by the Reynolds number only, but as we have shown[32] at the change of laminar flow it is necessary to consider the Ar number as well. What is considered to be the result of roughness, can be caused by different Ar numbers.

The structure of fixed beds can differ considerably from beds in the region of incipient fluidization and the roughness effect can be affected by the structure.

In our previous paper[5] we compared Eq. (*5*) with other relations and found a satisfactory agreement with equation of Leva[22] and Todes[7]. In Table VI are Eqs (*5*), (*7*) compared with Eqs (*38*), (*43*), (*46*) and with equation of Todes

$$\mathrm{Re}_p = \mathrm{Ar}/(1\,400 + 5.22\ \mathrm{Ar}^{0.5}) \tag{47}$$

and with equation[33]

$$\mathrm{Re}_p = \mathrm{Re}_t \frac{(10 + 0.275\ \mathrm{Ly})(250 + \mathrm{Ly})}{(990 + 2.850\ \mathrm{Ly})(140 + \mathrm{Ly})}\,, \tag{48}$$

which is valid for

$$\mathrm{Ly} = u_t^3 \varrho_f^2 / g\mu\varrho_s - \varrho_f) \leqq 6 \,.\, 10^5 \,, \tag{49}$$

where

$$\mathrm{Ly} = \mathrm{Re}_t^3/\mathrm{Ar} \,. \tag{50}$$

It can be seen from Table VI that Eq. (*5*) resp. (*7*) and (*38*) give relatively corresponding $\mathrm{Re_p}$ in the whole range of values of Ar number. This agreement is surprising when compared to data given in Table V and the only explanation is that deviations

TABLE VI

Comparison of the Best Equations for Calculation of $\mathrm{Re_p}$ of Monodisperse Non-Channelling Beds with Equations (*5*) and (*7*)

Ar	Values $\mathrm{Re_p}$ according to Equation					
	(*5*) resp. (*7*)	(*38*)	(*43*)	(*46*)	(*47*)	(*48*)
0·1	$9{\cdot}98 \,.\, 10^{-5}$	$9{\cdot}43 \,.\, 10^{-5}$	$6{\cdot}06 \,.\, 10^{-5}$	$8{\cdot}52 \,.\, 10^{-5}$	$7{\cdot}13 \,.\, 10^{-5}$	$1{\cdot}00 \,.\, 10^{-4}$
1	$9{\cdot}93 \,.\, 10^{-4}$	$9{\cdot}43 \,.\, 10^{-4}$	$6{\cdot}06 \,.\, 10^{-4}$	$8{\cdot}52 \,.\, 10^{-4}$	$7{\cdot}12 \,.\, 10^{-4}$	$1{\cdot}00 \,.\, 10^{-3}$
2	$1{\cdot}97 \,.\, 10^{-3}$	$1{\cdot}88 \,.\, 10^{-3}$	$1{\cdot}21 \,.\, 10^{-3}$	$1{\cdot}70 \,.\, 10^{-3}$	$1{\cdot}42 \,.\, 10^{-3}$	$2{\cdot}00 \,.\, 10^{-3}$
5	$4{\cdot}86 \,.\, 10^{-3}$	$4{\cdot}70 \,.\, 10^{-3}$	$1{\cdot}50 \,.\, 10^{-3}$	$4{\cdot}26 \,.\, 10^{-3}$	$3{\cdot}54 \,.\, 10^{-3}$	$5{\cdot}01 \,.\, 10^{-3}$
10	$9{\cdot}52 \,.\, 10^{-3}$	$9{\cdot}38 \,.\, 10^{-3}$	$6{\cdot}06 \,.\, 10^{-3}$	$8{\cdot}52 \,.\, 10^{-3}$	$7{\cdot}06 \,.\, 10^{-3}$	$9{\cdot}65 \,.\, 10^{-3}$
20	0·0184	0·0187	0·0121	0·0170	0·0141	0·0182
50	0·0434	0·0463	0·0303	0·0425	0·0348	0·0406
100	0·0817	0·0917	0·0605	0·0850	0·0689	0·0737
200	0·152	0·182	0·121	0·171	0·136	0·134
500	0·347	0·433	0·313	0·420	0·330	0·294
1 000	0·645	0·834	0·600	0·840	0·639	0·564
$2 \,.\, 10^3$	1·19	1·55	1·19	1·65	1·22	1·09
$5 \,.\, 10^3$	2·71	3·48	2·90	3·83	2·83	2·87
$1 \,.\, 10^4$	5·02	6·19	5·59	7·86	5·20	5·74
$2 \,.\, 10^4$	9·29	10·7	10·5	13·4	9·36	10·9
$5 \,.\, 10^4$	21·0	21·2	22·6	27·4	19·5	25·0
$1{\cdot}06 \,.\, 10^5$	41·0	36·1	40·2	60·9	34·2	42·5
$2 \,.\, 10^5$	60·0	55·8	62·7	70·5	53·6	64·4
$5 \,.\, 10^5$	104	102	113	123	98·2	114
$1 \,.\, 10^6$	158	160	171	182	151	169
$2 \,.\, 10^6$	240	248	254	284	228	226
$5 \,.\, 10^6$	417	438	419	436	382	398
$1 \,.\, 10^7$	633	670	606	626	558	552
$2 \,.\, 10^7$	960	1 020	870	896	808	844
$5 \,.\, 10^7$	1 660	1 780	1 390	1 430	1 305	1 200
$1 \,.\, 10^8$	2 530	2 690	1 990	2 030	1 910	1 670
$2 \,.\, 10^8$	3 840	4 080	2 820	2 880	2 704	2 330
$5 \,.\, 10^8$	6 670	7 040	4 480	4 600	4 280	3 620

from the hypothesis expressed by Eq. (*26*) are balanced by deviations of Eq. (*36*) from the correct dependence. It is not clear from the work by Wen and Yu[8] why they consider Eq. (*43*) to be more correct then Eq. (*38*).

Other equations give at $Ar > 1 \,.\, 10^7$ smaller Re_p values than Eqs (*5*) resp. (*7*). The differences substantially exceed the limiting error of our measurement.

The mutually corresponding Re_p and Ly numbers were determined in such a way that for the chosen Ar number was from data given by Perry[28] by use of the method from the papers[29] determined Re_t by interpolation formulas and from it $Ly = Re_t^3/Ar$. For values $Ar \geqq 5 \,.\, 10^6$ was found the power relation $Re_t = 2{\cdot}60342\ Ar^{0{\cdot}4782}$. Constants in Eq. (*43*) and (*46*) do not enable sufficiently accurate determination already at $Ar < 1000$. By repeating the procedure in derivation of these equations it was found that instead of 33·7, resp. 24·86 it is necessary to use values 33·673468 resp. 24·857143. Also the first terms under the exponent in these equations should be squares of these numbers.

Effect of the particle shape. If we define the characteristic length of a particle as a diameter of a sphere of equal volume then Eq. (*5*) is practically valid with the same accuracy for particles of different shape which do not form channelling beds.

Eq. (*7*) which is valid for transition and turbulent regions is independent on the shape of particles at such definition of the characteristic length only approximately. At the same Ar number spherical particles have the greatest value Re_p. Close to them are isometric and practically isometric shapes, and which one dimension increasing Re_p decreases, see Fig. 4. With the shapes we have used, the decrease was not less than 22·5%. With such tolerance, the Eq. (*7*) can be considered as valid for particles of various shapes.

Calculation, Comparison and Transformation of Empirical Equation for y

With regard to what was said above and to our papers[3,4], we have for calculation of y relations (*11*), resp. (*12*) and (*14*) from which we choose in accordance with relations (*13a*), resp. (*13b*).

One of possible[3,4] relations for y is equation

$$y = f(w_{pC}/w_{pF}\,,\ \bar{x}_F)\,. \tag{51}$$

If $w_{pC}/w_{pF} = z$, then, according to physical nature of y, the following boundary conditions can be formulated:

$$\text{for } z \rightarrow 1 \text{ is } y \rightarrow 0 \text{ for any } \bar{x}_F\,, \tag{52a}$$

$$\text{for } \bar{x}_F \rightarrow 0 \text{ is } y \rightarrow 0 \text{ for any } z\,. \tag{52b}$$

When $\bar{x}_F \to 1$ constant value of y dependent on z can be expected, such that even when the mixture contains only a small amount of coarse particles, its velocity at incipient fluidization is greater than it would be with only fine particles. This condition will be additionally verified.

Sufficiently accurate measurements of velocities at incipient fluidization of the mixture with small portion of coarse particles are very difficult and in our measurements we did not reach below $\bar{x}_C = 0{\cdot}04$, so that the case of $x_F \to 1$ must be considered as an extrapolation. We tried to ensure its correctness by the procedure at evaluating the experimental data.

To avoid the unfavourable affecting of the result by eventual invalidity of a hypothesis to which the polydisperse mixture is taken as a binary one as well as by an eventual effect of different flow character, we have used for determination of equation for y only the measurements with binary mixtures which comply with the condition $(1 - y)$ Ar $> 1{\cdot}06 \,.\, 10^5$. Values w_{pF}, resp. Re_{pF} were calculated by Eq. (5) resp. (9) and also by Eq. (7). Respective data are taken from one of our previous works[4]. At $\bar{x}_F =$ const. they are in agreement with relation

$$y = (z - 1)/(pz + q) \tag{53}$$

which at the same time fulfills the boundary conditions (52a) and (52b). Coefficients

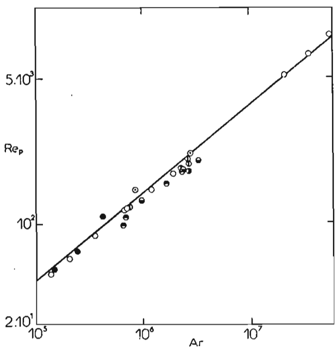

FIG 4

Deviations of Re_p Values of Non-Spherical Particles from Dependence $\mathrm{Re}_p = f(\mathrm{Ar})$ for Spherical Particles

Eq. (7); ○ limestone; ● sugar; ◑ barley; ⊙ electrotechnical beads; ⊖ rye; ⊖ wheat; ⦶ oats (a more detail characteristics of material used are given in the already published paper[5]).

p, q, are dependent on $\bar{x}_F$. With regard to experimental data for z = const. we can write

$$p = k_1/\bar{x}_F^a\,, \tag{54}$$

respectively

$$q = k_2/\bar{x}_F^a\,. \tag{55}$$

From that holds the empirical equation

$$y = k\bar{x}_F^a(z - 1)/(z - b)\,. \tag{56}$$

The following problems are met in determination of empirical constants a, b, and k:

Values Re_p were read off the plot $\log \Delta p = f(\log \mathrm{Re})$, therefore it can be assumed that their relative error approximately equals, *i.e.* that

$$\Delta\mathrm{Re}_p/\mathrm{Re}_p = C\,. \tag{57}$$

As for absolute errors holds

$$\Delta\mathrm{Re}_p = \frac{\mathrm{d}\,\mathrm{Re}_p}{\mathrm{d}y}\,\Delta y\,, \tag{58}$$

we get from Eqs (*12*) and (*14*)

$$\frac{\Delta y}{1 - y} = C_1\,. \tag{59}$$

In such case the method of the least squares requires a consideration of different weights of y values[34,35].

The weight of r-th measurement y_r is inversely proportional to the dispersion σ_r^2 which can be estimated by expression $(\Delta y_r)^2$, which according to Eq. (*59*) is

$$(\Delta y_r)^2 \sim (1 - y_r)^2\,. \tag{60}$$

After substituting from Eq. (*60*) for the weight of the r-th measurement and after linearization of Eq. (*56*) by expansion into the Taylor series, the values: $k = 1{\cdot}001$, $a = 0{\cdot}590$, $b = 0{\cdot}357$ were determined by successive approximation which corresponds to equation

$$y = \bar{x}_F^{0.59}(z - 1)/(z - 0{\cdot}357). \tag{61}$$

Let us verify whether Eq. (*61*) fulfills the condition $w_{pZ} > w_{pF}$. Eq. (*14*) can be transformed into the form

$$w_{pZ} = \{[w_{pC}^{2.661}(1 - \bar{x}_F^{0.59}) + w_{pF}w_{pC}^{1.661}(\bar{x}_F^{0.59} - 0{\cdot}357)]/(w_{pC} - 0{\cdot}357w_{pF})\}^{0.602} \tag{62}$$

if we substitute for y from Eq. (*61*) and simultaneously write according to definition

$$\mathrm{Re}_{pC} = \mathrm{Re}_{pCZ}(w_{pC}/w_{pZ})\,, \tag{63}$$

then from it according to Eq. (7) is

$$\mathrm{Ar_C} = (\mathrm{Re_{pCZ}} w_{pC}/0.03865 w_{pZ})^{1/0.602} . \tag{64}$$

From Eq. (62) for $\bar{x}_F \to 1$ we get

$$w_{pZ} = \{0.643 w_{pF} w^{0.661}/[1 - 0.357(w_{pF}/w_{pC})]\}^{0.602} . \tag{65}$$

Already with relatively small values of z ($z > 1$) the denominator in Eq. (65) can be taken equal to one and we get

$$(w_{pZ})_{\bar{x}_F \to 1} > 0.643^{0.602} w_{pF}^{0.602} w_{pC}^{0.398} . \tag{66}$$

If $w_{pZ} > w_{pF}$ then by substituting for w_{pZ} such quantity w_{pF} that the inequality holds

$$w_{pF} < 0.643^{0.602} w_{pF}^{0.602} w_{pC}^{0.398} \tag{67}$$

we obtain that the required condition $w_{pZ} > w_{pF}$ is fulfilled for all z for which holds the inequality

$$z = w_{pC}/w_{pF} > 1.95 . \tag{68}$$

This is qualitatively a good result because it is in fact valid for all beds with the segregation region.

In case that $(1 - y)\ \mathrm{Ar_C} < 1.06 \cdot 10^5$, but $\mathrm{Ar_C} > 1.06 \cdot 10^5$, *i.e.* $\mathrm{Re_{pCZ}}$ is calculated by Eq. (11), resp. Eq. (12), however, $\mathrm{Re_{pC}}$ is calculated by Eq. (7), we get

$$w_{pZ} = \frac{[w_{pC}(1 - x_F^{0.59}) + w_{pF}(x_F^{0.59} - 0.357)]^{0.602}\ w_{pCpC}^{0.398}}{3.304[1 - 0.357(w_{pF}/w_{pC})]^{0.602}\ \mathrm{Re}_{pCZ}^{(0.602-n)/n}} . \tag{69}$$

From that for $\bar{x}_F \to 1$ at $n = 0.89$ and $\mathrm{Re_{pCZ}} = 41$ the condition $w_{pZ} > w_{pF}$ is fulfilled when

$$z > 3 . \tag{70}$$

With decreasing value of $\mathrm{Re_{pCZ}}$ the critical value z increases inversely to $\mathrm{Re}_{pCZ}^{0.814}$. At the condition $(1 - y)\ \mathrm{Ar_C} < 1.06 \cdot 10^5$ the value $\mathrm{Re_{pZ}} = 41.0$ is the maximum attainable one and therefore the condition (70) cannot be considered as satisfactory. Even less satisfactory result is obtained for $(1 - y)\ \mathrm{Ar_C} < 1.06 \cdot 10^5$; $\mathrm{Ar_C} < 1.06 \cdot 10^5$, when the transformed equation is

$$w_{pZ} = \left\{\frac{w_{pC}^{1+1/n}(1 - \bar{x}_F^{0.59}) + w_{pF} w_{pC}^{1/n}(\bar{x}_F^{0.59} - 0.357)}{w_{pC} - 0.357 w_{pF}}\right\}^n . \tag{71}$$

The inequality $w_{pZ} > w_{pF}$ at $\bar{x}_F \to 1$ requires

$$z > 1/0.643^{n/(1-n)} \tag{72}$$

which already at $n = 0.89$ gives very large values z. The large exponent $n/(1 - n)$ suggests that instead of the value 0.643 in Eq. (72) should be the value close to one, *i.e.* if Eq. (56) is to be valid then $b = 0$, resp. $k_2 = 0$. Generalization: For $(1 - y)\ \mathrm{Ar_C} > 1.06 \cdot 10^5$, resp. $\mathrm{Re_{pCZ}} > 41.0$ and $\mathrm{Ar_C} > 1.06 \cdot 10^5$, resp. $\mathrm{Re_{pC}} > 41.0$, b is constant $b = 0.357$. However, if $(1 - y)\ \mathrm{Ar_C} < 1.06 \cdot 10^5$ resp. $\mathrm{Re_{pCZ}} < 41.0$, then b decreases with the decreasing value $\mathrm{Re_{pCZ}}$ and at $\mathrm{Ar_C} < 1.06 \cdot 10^5$ it attains the zero value.

For deviations in Re_{pZ} numbers which are caused by the change of constant b, the ratio is characteristic

$$Y = \frac{1 - (z-1)/(z-0.357)}{1-(z-1)/z} = \frac{0.643z}{z-0.357} \tag{73}$$

which changes in the interval $Y \in \langle 0.643; 1 \rangle$ when $z \in \langle 1, \infty \rangle$. If we use at $(1-y)\,Ar_C < 1.06 \,.\, 10^5$ $b = 0.357$ instead of $b = 0$, at $n = 0.89$, the ratio of Reynolds numbers will be

$$\frac{(Re_p)_{b=0.357}}{(Re_p)_{b=0}} = Y^{0.89}, \tag{74}$$

where $Y^{0,89} \in \langle 0.675, 1 \rangle$ and already at $z = 2$ it attains the value 0·805, *i.e.* the calculated Re numbers will be by 20 to 30% smaller than at $b = 0$.

When we want to avoid these errors, we use the following instruction: If $(1-y)\,Ar_C > 1.06 \,.\, 10^5$, resp. $Re_{pCZ} > 41.0$ and simultaneously $Ar_C > 1.06 \,.\, 10^5$, resp. $Re_{pCZ} > 41.0$, we choose

$$y = \bar{x}_F^{0.59}(z-1)/(z-0.357)\,. \tag{75}$$

At

$$(1-y)\,Ar_C < 1.06 \,.\, 10^5\,, \quad \text{resp.} \quad Re_{pCZ} < 41.0$$

and simultaneously

$$Ar_C > 1.06 \,.\, 10^5\,, \quad \text{resp.} \quad Re_{pC} > 41.0\,,$$

then

$$y = (1/2)\,\bar{x}_F^{0.59}[(z-1)/(z-0.357) + (z-1)/z]\,. \tag{76}$$

If the relation is valid

$$(1-y)\,Ar_C < 1.06 \,.\, 10^5\,, \quad \text{resp.} \quad Re_{pCZ} < 41.0$$

and simultaneously

$$Ar_C < 1.06 \,.\, 10^5\,, \quad \text{resp.} \quad Re_{pC} < 41.0\,,$$

then

$$y = \bar{x}_F^{0.59}(z-1)/z\,. \tag{77}$$

Experimental data are in agreement with these conclusions where it is at the same time assumed that the power $\bar{x}_F^{0.59}$ remains unchanged regardless of values $(1-y)\,Ar_C$, resp. Ar_C within the whole studied range

$$(1-y)\,Ar_C \in \langle 1.9 \,.\, 10^3\,, 1.12 \,.\, 10^7 \rangle$$

and simultaneously

$$Ar_C \in \langle 3.9 \,.\, 10^3\,, 4.57 \,.\, 10^7 \rangle\,,$$

resp.

$$Re_{pCZ} \in \langle 0.845\,; 10^3 \rangle\,.$$

Transformation of calculation equations for y. Eq. (*61*), resp. (*76*) and (*77*) are advantageous for calculation of y only if the values w_{pC} and w_{pF} are given in advance. However, more often the values d_C and d_F are given. As we have shown before[3], y can be calculated from criterion equation

$$y = f_5(d_C/d_F, \mathrm{Ar}_C, \bar{x}_F), \tag{78}$$

$$y = f_6(w_{pC}/w_{pF}, \mathrm{Ly}_C, \bar{x}_F), \tag{79}$$

$$y = f_7(u_{tC}/u_{tF}, \mathrm{Ly}_{tC}, \bar{x}_F), \tag{80}$$

where the variable Ly_C does not practically affect the function f_6 (better estimation of this effect is given in our comments on Eqs (*76*) and (*77*)). We have made use of the fact that Eq. (*79*) can be expressed with sufficient accuracy by relation (*51*) for basical evaluation of data.

If we express w_{pC} and w_{pF} by the use of Eq. (*7*) or (*9*), relation (*79*) is transformed into Eq. (*78*). But in this case it is necessary to take into account that the relation holds

$$z = \frac{w_{pC}}{w_{pF}} = \frac{\mathrm{Re}_{pC}}{\mathrm{Re}_{pF}} \frac{d_F}{d_C} \tag{81}$$

and simultaneously

$$\mathrm{Ar}_F = \mathrm{Ar}_C \, d_F^3/d_C^3 \, . \tag{82}$$

Obviously, there must be distinguished three cases limited by the following conditions:

a)

$$\mathrm{Ar}_C > 1{\cdot}06 \, . \, 10^5 \, , \quad \text{resp.} \quad \mathrm{Re}_{pC} > 41{\cdot}0 \, , \tag{83}$$

$$\mathrm{Ar}_F > 1{\cdot}06 \, . \, 10^5 \, , \quad \text{resp.} \quad \mathrm{Re}_{pF} > 41{\cdot}0 \, ,$$

when

$$z = (d_C/d_F)^{0{\cdot}806} \, . \tag{84}$$

If relation (*83*) is valid, then Eq. (*61*) is valid as well because $\mathrm{Re}_{pCz} > \mathrm{Re}_{pF}$, resp. $(1 - y) \, \mathrm{Ar}_C > 1{\cdot}06 \, . \, 10^5$. After substitution into relation (*61*) we get:

$$y = \bar{x}_F^{0.59} \, \frac{(d_C/d_F)^{0{\cdot}806} - 1}{(d_C/d_F)^{0{\cdot}806} - 0{\cdot}357} \, . \tag{61a}$$

Under conditions (*83*) the quantity y is independent of Ar_C.

b)

$$\mathrm{Ar}_C < 1{\cdot}06 \, . \, 10^5 \, , \quad \text{resp.} \quad \mathrm{Re}_{pC} < 41{\cdot}0 \, , \tag{85}$$

when

$$z = 0{\cdot}000611^{(n_C - n_F)} \, \mathrm{Ar}_C^{(n_C - n_F)} \, (d_C/d_F)^{3n_F - 1} \, , \tag{86}$$

where n_C, resp. n_F are read off the graph of Fig. 1 for $\mathrm{Ar}_C < 200$, resp. $\mathrm{Ar}_F < 200$.

At conditions (*85*) Eq. (*77*) is valid, from which we get

$$y = \bar{x}_F^{0.59} \frac{0.000611^{(n_C - n_F)} \, \mathrm{Ar_C}^{(n_C - n_F)} \, (d_C/d_F)^{3n_F - 1} - 1}{0.000611^{(n_C - n_F)} \, \mathrm{Ar_C}^{(n_C - n_F)} \, (d_C/d_F)^{3n_F - 1}} \,. \tag{87}$$

However, most frequent is $\mathrm{Ar_C} > 200$, $\mathrm{Ar_F} > 200$ when $n_C = n_F = 0.89$ and thus

$$y = \bar{x}_F^{0.59} \frac{(d_C/d_F)^{1.67} - 1}{(d_C/d_F)^{1.67}} \,. \tag{77a}$$

c)

$$\mathrm{Ar_C} > 1.06 \,.\, 10^5 \,, \quad \text{resp.} \quad \mathrm{Re_{pC}} > 41.0 \,, \tag{88}$$

$$\mathrm{Ar_F} < 1.06 \,.\, 10^5 \,, \quad \text{resp.} \quad \mathrm{Re_{pF}} < 41.0 \,,$$

when

$$z = [(0.03865)/(0.000611^{n_F})] \, [\mathrm{Ar_C}^{(0.602 - n_F)} \, (d_C/d_F)^{3n_F - 1}] \,. \tag{89}$$

If simultaneously with conditions (*88*) the inequality holds

$$(1 - y) \, \mathrm{Ar_C} > 1.06 \,.\, 10^5 \,, \quad \text{resp.} \quad \mathrm{Re_{pCZ}} > 41.0 \,, \tag{89a}$$

then the relation (*61*) must be transformed, which gives

$$y = \bar{x}_F^{0.59} \frac{[(0.03865)/(0.000611^{n_F})] \, [\mathrm{Ar_C}^{(0.602 - n_F)} \, (d_C/d_F)^{3n_F - 1}] - 1}{[(0.03865)/(0.000611^{n_F})] \, [\mathrm{Ar_C}^{(0.602 - n_F)} \, (d_C/d_F)^{3n_F - 1}] - 0.357} \,. \tag{61b}$$

Eq. (*61b*) at $n_F = 0.89$ has the form

$$y = \bar{x}_F^{0.59} \frac{28.0 \, \mathrm{Ar_C}^{-0.288} \, (d_C/d_F)^{1.67} - 1}{28.0 \, \mathrm{Ar_C}^{-0.288} \, (d_C/d_F)^{1.67} - 0.357} \,. \tag{61c}$$

However, if simultaneously with conditions (*88*) holds

$$(1 - y) \, \mathrm{Ar_C} < 1.06 \,.\, 10^5 \,, \quad \text{resp.} \quad \mathrm{Re_{pCZ}} < 41.0$$

then the relation (*76*) must be transformed into

$$y = (1/2) \, \bar{x}_F^{0.59} [(X_1 - 1)/(X_1 - 0.357) + (X_1 - 1)/X_1] \,, \tag{76a}$$

where

$$X_1 = [(0.03865)/(0.000611)^{n_F}] \, [\mathrm{Ar_C^{(0.602 - n_F)}} (d_C/d_F)^{3n_F - 1}] \,, \tag{90}$$

respectively, at $n_F = 0.89$

$$X_1 = 28.0 \mathrm{Ar_C^{-0.288}} \, (d_C/\mathrm{d_F})^{1.67} \,. \tag{91}$$

Limitation of validity of equation for calculation of y. The validity of equations defining y at very large Ar_C numbers can be determined neither from the model nor from the experimental data. The maximum verified value was $Ar_C = 4{\cdot}57 \,.\, 10^7$ respectively, $Re_{pCZ} = 1000$.

In cases where before reaching the velocity at incipient fluidization an intensive carryover of some of the fractions from the column takes place, there can be expected a correction in definition of the fraction of fine particles which was given by relation (*4b*). The entrainment of particles intensively carried from the unit can dampen the fluctuation of velocity and pressure above the last stationary layer of coarse particles; the value y can be, therefore, greater than corresponds to the bed composition. In other words, we expect that for fractions intensively entrained the actual mass fractions according to definition d_F in Eq. (*46*) cannot be used. Further experimental data are necessary for determination of the respective corrections.

We demonstrate how to calculate whether before reaching the velocity at incipient fluidization of the mixture the carryover of particles from the column occurs. We consider the case when holds

$$(1 - y)\,Ar_C > 1{\cdot}06 \,.\, 10^5\,, \quad \text{resp.} \quad Re_{pCZ} > 41{\cdot}0\,, \tag{92}$$

i.e. when Re_{pCZ} is calculated by relation (*14*).

The particles with the greatest terminal velocity u_{tk} will be at the velocity at incipient fluidization of the mixture w_p entrained in accordance with the relation

$$u_{tk} = \alpha w_p\,, \tag{93}$$

where $\alpha > 1$ and is dependent on the Reynolds number $Re = w_p D \varrho_f / \mu$, column height, bed height *etc.* So far, we are able to determine the value α only in some cases[35]. Into entrainment get particles of all sizes $d^* \leqq d_k$, for the terminal velocities of which holds

$$u_t \leqq \alpha u_{tk}\,. \tag{93a}$$

With increasing difference between the quantities u_t and αw_p with respect to inequality $u_t < \alpha w_p$ the entrainment intensity of fraction with terminal velocity u_t increases.

With regard to relations (*14*) and (*93a*) as a condition for the maximum velocity without entrainment of a fraction with terminal velocity u_t the inequality can be written

$$u_t > (0{\cdot}03865 \alpha \mu [(1 - y)\,Ar_C]^{0{\cdot}602}) / \varrho_f d_C\,. \tag{94}$$

Substitution of equality in relation (*94*) defines the fraction with terminal velocity u_{tk}, *i.e.* such fraction which at the velocity at incipient fluidization of the mixture just begins to take part in entrainment.

To the following inequality

$$u_t < (0.03865\alpha\mu[(1 - y)\,\mathrm{Ar_C}]^{0.602})/\varrho_f d_C \tag{95}$$

correspond fractions with terminal velocity u_t entrained at velocity at incipient fluidization if entrainment would not affect the velocity at incipient fluidization of the mixture.

For inequalities

$$(1 - y)\,\mathrm{Ar_C} < 1.06\,.\,10^5\,, \quad \text{resp.} \quad \mathrm{Re_{pCZ}} < 41.0\,, \quad \text{is} \quad \mathrm{Re_{pCZ}}$$

calculated from Eq. (*12*) and we get

$$u_t \gtreqless \{\alpha\mu[0.000611(1 - y)\,\mathrm{Ar_C}]^n\}/\varrho_f d_C\,, \tag{96}$$

where significance of the signs of inequalities and equalities is the same as before.

The standardized quantity $\mathrm{Re_{p\,calc}}/\mathrm{Re_{p\,exp}}$ was used as a suitable measure for considering the correctness of calculation relations in individual cases. For its theoretical mean value holds $E(\mathrm{Re_{p\,calc.}}/\mathrm{Re_{p\,exp}}) = 1$. The standard deviation of this standardized quantity is then a general characteristic accuracy of calculation relations. Its value $s = 0.178$ was determined from the system of our experimental data. Since the theoretical mean value is equal to one, the standard deviation 0·178 represents at the same time the variation coefficient. The value of variation coefficient 0·178 is rather large but with respect to the accuracy of measurements of such parameters as the characteristic length of individual fractions, reproducibility of porosity, method of determination of the velocity at incipient fluidization and the approximate validity of the hypothesis on representative size of the fraction of fine particles, we take it as acceptable.

For a more complete characterization of our experimental material and of equations we wish to add that from 114 data, a mean deviation of $\pm 5\%$, $\pm 10\%$, $\pm 15\%$, $\pm 20\%$, have accordingly 25·4%, 48·2%, 63·1%, and 78·0% of the data. The maximum measured deviation was $\mathrm{Re_{p\,calc.}}/\mathrm{Re_{p\,exp.}} = 1.461$ for values of arguments: $\bar{x}_F = 0.7$, $z = 2.536$, when $y_{exp} = 0.771$ and $y_{calc} = 0.571$. As more significant we consider the deviations above $\pm 30\%$. With such error of measurement are affected data No 43 and No 55 in our previous work[4]. Data corresponding to numbers 81 to 83 and 95 to 106, where $\mathrm{Re_{p\,calc}}/\mathrm{Re_{p\,exp}} \in \langle 0.596 : 0.732 \rangle$ correspond to coarse particles containing a greater amount of a narrow fraction having the characteristic length $d_i \approx 0.5\,d_{max}$. Such fraction is according to Eq. (*3*) considered the smallest fraction of coarse particles. If it is added to fine particles, then the results are significantly better. This means that value 0·5 in Eq. (*3*) should be greater, which in agreement with our experiments we estimate to be in the range 0·57 to 0·60 *i.e.* instead of Eq. (*3*) $a\,d_{max} \leqq d_i \leqq d_{max}$ should be written, where $0.57 \leqq a \leqq 0.60$.

LIST OF SYMBOLS

a coefficient
$Ar = gd^3(\varrho_s - \varrho_f)\,\varrho_f/\mu^2$ Archimedes number
Ar^* Archimedes number defined by Eq. (*34*)
Ar_C, Ar_F Archimedes number with the characteristic length d_C, respectively d_F
C index characterizing coarse particles
d characteristic length of the particle according to Eq. (*15*)
d_i characteristic length of so-called narrow fraction of particles in which the range of sizes is not greater than the value limited by two consecutive sieves DIN 1210 or Tyller.
$d_i = \Sigma n_i d_{is}^3/\Sigma n_i d_{is}^2$ for spherical particles
$d_i = 6m/(\pi N \varrho_s)^{1/3}$ for non-spherical particles
d_{iC}, d_{iF} value of d_i in that part of the bed which is denoted as the fraction (wide) of coarse particles resp. fine particles
d_C, d_F representative (effective) diameter of fraction (wide) coarse resp. fine particles (Eq. (*4a*) resp. (*4b*))
d_{is} diameter of spherical particle
F_0 hydrodynamic force by which at superficial velocity w the liquid acts on the particles in the fluidized bed
F_s hydrodynamic force by which, at other conditions same, as that to which is related F_0, the liquid would act on one particle freely moving in unlimited liquid by velocity w
$f(\varepsilon)$ function defined by Eq. (*26*)
g gravitational acceleration
k, k_1, k_2 proportionality coefficients
l bed height
$Ly = Re_t^3/Ar$ Ljaščenko number
Ly_C, Ly_{tC} Ly number, if $u_t = w_{pC}$, resp. $u_t = u_{tC}$
m mass of N particles at determination of d_i
n exponent in Eq. (*9*), defined by Eq. (*10*)
n_i number of particles of diameter d_{is}
N number of particle (minimum 200) at determination of d_i
Δp pressure drop in the bed
Δp_t value of Δp equal to the effective particle weight on unit of cross-sectional area (constant diameter of the column)
Δp^* surplus drop of static pressure in so-called last fixed bed of coarse particles at the velocity at incipient fluidization caused by velocity pulsation
$Re = wd\varrho_f/\mu$ Reynolds number
Re_p Reynolds number when $w = w_p$
Re_{pC} Re number when $w = w_{pC}$, and $d = d_C$
Re_{pF} Re number when $w = w_{pF}$, and $d = d_F$
Re_{pCZ} Re number when $w = w_{pZ}$, and $d = d_C$
$Re_{p\,calc}$ calculated value of Re_{pCZ}
$Re_{p\,exp}$ experimental value of Re_{pCZ}
Re_t Re number when $w = u_t$
u_t terminal velocity of particle, *i.e.* the final velocity of particle freely falling at the earth gravitation in unlimited real liquid
u_{tC}, u_{tF} value of u_t corresponding to d_C, resp. d_F

w superficial velocity of liquid in the column (volumetric flow rate divided by the cross-sectional area of the column)
w_p value of w at the incipient fluidization, *i.e.* the velocity at incipient fluidization (minimum-fluidization velocity)
w_{pC}, w_{pF} value of w_p corresponding to monodisperse fraction d_C, resp. d_F at other conditions same as w_{pZ}
w_{pZ} velocity at incipient fluidization (w_p) of the mixture of coarse and fine particles (polydispersion bed)
$\bar{x}_{iC}$ mass fraction of (narrow) fraction d_{iC} in coarse particles
$\bar{x}_{iF}$ mass fraction of (narrow) fraction d_{iF} in fine particles
$\bar{x}_F$ mass fraction of fine particles (wide fraction) in the bed
y function expressing the effect of pressure and velocity fluctuation caused by the fluidized part of polydisperse bed above the last fixed bed
Y quantity defined by Eq. (*73*)
$z = w_{pC}/w_{pF}$
ϱ_f gas density
ϱ_s density of particles
ε bed porosity
ε_p porozity at velocity at incipient fluidization (value ε at $w = w_p$)
μ gas viscosity
φ shape factor defined as sphericity, *i.e.* as the ratio of surface of a sphere of the same volume as the particle to the particle surface
$\xi = (4/3)\,gd(\varrho_s - \varrho_f)/w^2\varrho_f$ friction factor for particle in the bed
$\xi_t = (4/3)\,gd(\varrho_s - \varrho_f)/u_t^2\varrho_f$ friction factor for freely falling particle with terminal velocity u_t
$\xi_s = \xi/f(\varepsilon)$ friction factor

REFERENCES

1. Beňa J., Ilavský J.: Chem. zvesti *21*, 401 (1967).
2. Ilavský J., Beňa J.: Chem. zvesti *21*, 877 (1967).
3. Beňa J., Havalda I., Bafrnec M., Ilavský J.: This Journal *33*, 2620 (1968).
4. Beňa J., Havalda I., Ilavský J., Bafrnec M.: This Journal *33*, 2833 (1968).
5. Beňa J., Ilavský J., Kossaczký E., Valtýni J.: This Journal *28*, 555 (1963).
6. Romankov P. G., Raškovskaja N. B.: *Suška v Kipjaščem Sloje*. Chimija, Moscow 1964.
7. Aerov M. E., Todes O. M.: *Gidravličeskije i Teplovyje Osnovy Raboty Apparatov so Stacionarnym i Kipjaščim Zernistym Slojem*. Chimija, Leningrad 1968.
8. Wen C. Y., Yu Y. H.: Chem. Eng. Progr., Symp. Ser. *62*, 100 (1966).
9. Beňa J., Ilavský J., Kossaczký E., Zakutný O.: Chem. zvesti *13*, 170 (1959).
10. Beňa J., Ilavský J., Kossaczký E., Neužil Ľ.: This Journal *28*, 293 (1963).
11. Wilhelm R. H., Kwauk M.: Chem. Eng. Progr. *44*, 201 (1948).
12. Ergun S., Orning A. A.: Ind. Eng. Chem. *41*, 1179 (1949).
13. Lewis W. K., Gilliland E. R., Bauer W. C.: Ind. Eng. Chem. *41*, 1104 (1949).
14. Baerg A., Klassen J., Gisher P. E.: Can. J. Res. *28*, 287 (1950).
15. Agarwal O. P., Storrow J. A.: Chem. Ind. (London) *1951*, 278.
16. Furukawa J., Ohmae T., Ueki I.: Chem. High Polymers (Tokyo) *8*, 111 (1951).
17. Miller C. O., Longwinuk A. K.: Ind. Eng. Chem. *43*, 1220 (1951).
18. Heerden van C., Nobel A. P. P., Krevelen van D. W.: Chem. Eng. Sci. *1*, 37 (1951).
19. Lewa M., Weintraub M., Grummer M., Pollchik M., Storch H. H.: U.S. Bur. Mines Bull. *1951*, 504.

20. Yagi S., Muchi I., Aochi T.: Chem. Eng. (Japan) *16*, 307 (1952).
21. Shirai T.: *Fixed Bed, Fluidized Bed, and the Fluid Resistance*; *Heat Transfer and Mass Transfer of Single Particle.* Research Rept., Tokyo Inst. Technol. Japan, Tokyo 1954.
22. Leva M., Shirai T., Wen C. Y.: Genie Chim. *75*, 33 (1956).
23. Johanson L. N.: Unclassified Rept. HW — 52891, General Electric Co., Hanford Div. 1957.
24. Kelly V. P.: *Thesis.* University Idaho, Moscow 1958.
25. Fetterman C. P.: *Thesis.* University Washington, Seattle 1958.
26. Fan L. T., Schwartz C. J.: Can. J. Chem. Eng. *37*, 204 (1959).
27. Narsimhan G.: AIChE J. *11*, 550 (1965).
28. Perry R. H., Chilton C. H., Kirkpatrick S. D.: *Chemical Enginners' Handbook.* McGraw-Hill, New York 1963.
29. Beňa J., Lodes A., Ševčík J.: Chem. průmysl *19*, 291 (1969).
30. Ergun S.: Chem. Eng. Progr. *48*, 227 (1952).
31. Denton W. H.: *General Discussion on Heat Transfer*, ASME, 1951, p. 370; Repts. At. Energy, Res. Est. No E/R 1095 (1957).
32. Beňa J.: Chem. průmysl *8*, 516 (1958).
33. Beránek J., Sokol D., Winterstein G.: *Wirbelschichttechnik*, Leipzig 1964.
34. Linnik Ju. V.: *Metod Najmenšich Kvadratov i Osnovy Mat.- Statističeskoj Obrabotky Nabl'udenij.* Fizmatgiz, Moscow 1962.
35. Beňa J., Lodes A., Šefčík J.: Chem. průmysl *18*, 539 (1968).

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